RIMS Workshop (Type A)
Japan-Austria workshop on discrete and global surface theory
September 14 – 16, 2026, at RIMS, Kyoto University

Abstract

September 14 (Mon)

13:00–14:00

Udo Hertrich-Jeromin (Vienna University of Technology)

Linear Weingarten surfaces

We say that a surface is linear Weingarten if its Gauss and mean curvatures satisfy a non-trivial affine relation — in space forms, it is irrelevant whether we consider the intrinsic or extrinsic Gauss curvature. These surfaces have made a recurrent appearance in our Japan/Austria (in fact, also Japan/pre-Austria) joint work and projects. The first part of the talk will aim to motivate the interest in linear Weingarten surfaces by discussing their appearances, mainly in collaborative work, and the contexts of those appearances. The second part of the talk will focus on a current enquiry about a particular class of linear Weingarten surfaces: constant Gauss curvature surfaces of revolution in space forms. This is joint research with S Bentrifa, M Kokubu and D Polly.

14:15–15:15

Keita Takahashi (Institute of Science Tokyo)

Completeness conditions for globally hyperbolic spacetimes

In this talk, we discuss several completeness conditions for globally hyperbolic spacetimes within the framework of Lorentzian length spaces and spacetimes with low-regularity metrics. These conditions, originally introduced by Busemann and Beem, serve as Lorentzian analogues to those found in the classical Hopf-Rinow theorem. We highlight recent results showing that these conditions ensure the metric completeness of the space of Cauchy sets, as established by Lange and Peteranderl. Furthermore, we explore several metric properties of this space.

Yuta Yamauchi (Yokohama National University)

Minimal total absolute curvature for equiaffine immersions

For immersions of compact n-dimensional manifolds into Euclidean space, the total absolute curvature is a global geometric quantity defined by integrating the absolute value of the Lipschitz–Killing curvature. The Chern–Lashof theorem states that the total absolute curvature is bounded below by the sum of the Betti numbers. Moreover, it is equal to 2 if and only if the image is a convex hypersurface embedded in an $(n+1)$-dimensional affine subspace. In 2001, Koike introduced the total absolute curvature for equiaffine immersions of arbitrary dimension and codimension, and established a Chern–Lashof type inequality. However, a geometric characterization of the case in which the total absolute curvature attains its minimum

value had remained unknown in the equiaffine setting. In this talk, we investigate the relationship between the minimality of the total absolute curvature and convexity for equiaffine immersions, without assuming the non-degeneracy of the affine fundamental form. We prove that the total absolute curvature is equal to 2 if and only if the image is a convex hypersurface embedded in an $(n+1)$-dimensional affine subspace.

Riku Kishida (Institute of Science Tokyo)

Zero mean curvature surfaces with singularities in the isotropic 3-space

The isotropic 3-space $\mathbb{I}^3$ is the 3-dimensional real vector space with the degenerate inner product of signature $(0++)$. For a space-like surface in $\mathbb{I}^3$, the mean curvature function can be defined in a natural way. A space-like surface whose mean curvature function vanishes everywhere is called a zero mean curvature surface. A zero mean curvature surface in $\mathbb{I}^3$ admits a Weierstrass-type representation formula. In this talk, we introduce a class of zero mean curvature surfaces with singularities in $\mathbb{I}^3$, which we call ZMC-faces. Using this framework, we show that several Osserman-type inequalities can be obtained under certain assumptions on both completeness and finiteness of the total curvature. We also present some examples which attain the equality in these inequalities.

15:45–16:45

Francis Burstall (University of Bath)

Isothermic tori with spherical curvature lines

I will make a conformally invariant analysis of isothermic surfaces with spherical curvature lines in one or both families. I will show how these are constructed from particular holomorphic maps to S^2 (those of finite type with spectral genus one) and indicate how this construction provides examples of isothermic tori. In so doing, we provide a complete generalisation of the results of Bobenko–Hoffmann–Sageman-Furnas on isothermic tori with planar curvature lines to the spherical case.

This is joint work with Cho, Pember and Szewieczek.

September 15 (Tue)

9:00–9:30

Masaya Hara (National Institute of Technology, Anan College)

Decomposition of zero mean curvature surfaces

In this talk, we introduce a decomposition theorem for zero mean curvature surfaces in Euclidean, Minkowski, and isotropic 3-spaces. In particular, we prove that every non-planar minimal surface in Euclidean space can be uniquely decomposed as the sum of two zero mean curvature surfaces in isotropic space, and discuss geometric properties that are preserved under this decomposition. This talk is based on joint work with Shintaro Akamine (Nihon University), Joseph Cho (Handong Global University), and Yuta Ogata (Kyoto Sangyo University).

9:40–10:40

Joseph Cho (Handong Global University)

Point of sphere geometries

Many interesting surface classes in Minkowski 3-space including maximal surfaces and constant mean curvature surfaces are known to admit non-degenerate singularities. These singularities differ from the isolated singularities appearing on analogous surfaces in Euclidean 3-space. However, the case of constant mean curvature surfaces shows that some surfaces in Minkowski 3-space also appear with disconnected components, admitting blow-up points. In this talk, we propose Laguerre geometry to gain insight into the fundamental difference between Euclidean 3-space and Minkowski 3-space, and to obtain an intuitive understanding of why non-degenerate singularities and blow-up points appear on surfaces in Minkowski 3-space, with the case of maximal surfaces serving as a useful case study.

10:50–11:30

Sara Bentrifa (Vienna University of Technology)

Analytic Extension of Rotational CHMC Surfaces in Hyperbolic Space

We study the analytic continuation of linear Weingarten surfaces across the ideal boundary of hyperbolic space. Focusing on rotational surfaces of constant harmonic mean curvature, we establish the analytic continuation of the surface across the ideal boundary via the extension of its Gauss map. Geometrically, this continuation corresponds to a 180-degree rotation of the profile curve around the boundary curve.

Naoya Suda (Kobe University)

Algebraic construction of discrete spacelike K-surfaces in Minkowski 3-space

For discrete surfaces of constant negative Gaussian curvature, Bobenko–Pinkall and Hoffmann–Sageman-Furnas gave algebraic constructions in terms of asymptotic and curvature line coordinates, respectively. In this talk, we apply their algebraic constructions to discrete spacelike surfaces with constant negative extrinsic Gaussian curvature in Minkowski space. We also discuss the relationship with discrete timelike surfaces.

13:00–14:00

Franz Pedit (University of Massachusetts Amherst)

Higher genus minimal Lagrangian surfaces in CP^2

I will explain how to construct higher genus compact minimal Lagrangian surfaces in CP^2 whose Legendrian lifts are the first examples of embedded compact special Legendrian surfaces in S^5 of genus greater than one. The surfaces constructed have the conformal structure of the Fermat curve of degree k and genus $1/2(k-1)(k-2)$. Our approach combines an elementary implicit function argument with the description of minimal Lagrangian surfaces via loop algebra-valued meromorphic connections and a characterization of the unitarizability locus in the $SL_3(C)$-character variety of the thrice-punctured sphere. This is joint work with Sebastian Heller and Charles Ouyang.

14:15–15:15

Wayne Rossman (Kobe University)

Maximal Darboux transformations, with a bit of history

After giving an outline of the experiences and background underlying this topic, I will talk about Darboux transformations between spacelike maximal surfaces. Such surfaces generally have singularities when considered globally, so we wish to extend the notion of Darboux transform to include singularities, and we then describe how singularities can (and cannot) transform. This is joint work with J. Cho, M. Hara, A. Honda and T. Raujouan.

15:45–16:45

Masaaki Umehara (Institute of Science Tokyo)

The geometry of constant mean curvature and constant curvature surfaces: Encounters with Wayne and a bit of history

In this talk, I will review the history of my encounters and collaborations with Professor Rossman, and discuss surfaces of constant mean curvature one in hyperbolic 3-space, flat surfaces in hyperbolic 3-space and their singularities, as well as space-like maximal surfaces in Minkowski 3-space and mixed-type entire graphs arising as their analytic extensions.

September 16 (Wed)

9:50–10:30

Kaoru Iino (Yokohama National University)

Intrinsic invariants of $(3,4)$*-cuspidal edges*

$(3,4)$-cuspidal edges are important singularities that arise naturally on parallel surfaces of regular surfaces, such as ellipsoids. For ordinary cuspidal edges, Saji–Umehara–Yamada introduced the singular curvature as an intrinsic invariant of cuspidal edges, and showed, in particular, that its sign describes the local convexity of cuspidal edges. On the other hand, for $(3,4)$-cuspidal edges, the sign of the singular curvature is not canonically well-defined, so different invariants are needed. In this talk, we introduce two intrinsic invariants for 3-type edges, a class of singularities that includes $(3,4)$- and $(3,5)$-cuspidal edges. We show that these invariants describe the local convexity of $(3,4)$-cuspidal edges. Moreover, using these invariants, we give a necessary and sufficient condition for the boundedness of the Gaussian curvature. We also obtain intrinsic criteria for generic $(3,4)$- and $(3,5)$-cuspidal edges.

Sakuma Takeshita (Tokushima University)

Iterated minimal Darboux transformations

Transformations for surfaces are crucially important in constructing new surfaces. Here we focus on isothermic surfaces. They admit conformal curvature line coordinates, and it is known that they admit various transformations.

In this presentation, we introduce the permutability of transformations for isothermic surfaces in the Euclidean space. We show that a superposition principle of Darboux transformations

for minimal surfaces can be viewed as an application of the permutability of transformations for isothermic surfaces. Furthermore, we illustrate concrete examples of minimal surfaces obtained by our result, and explore their asymptotic behaviors. This is joint work with Masashi Yasumoto.

10:45–11:15

Denis Polly (Vienna University of Technology)

Discrete surfaces inspired by the Ising model

The Ising model is probably one of the most studied models in statistical mechanics. In recent years discrete geometric methods, in particular incircular nets, have been utilized to study its thermodynamic limit: surprisingly, certain lifts of incircular nets in Lorentz space converge to a maximal surface as the model approaches its limit. In this talk, we describe how a subclass of incircular nets corresponds to maximal surfaces in the discrete category. Based on joint work with N. Affolter, F. Dellinger, Ch. Müller and Nina Smeenk.

11:30–12:00

Masatoshi Kokubu (Tokyo Denki University)

Two metrics on an ambient space and constant Gaussian curvature surfaces they induce

We can observe a somewhat surprising phenomenon that some constant Gaussian curvature surfaces with respect to two different metrics on the same ambient space share the same profile curve. Based on this phenomenon, we shall try to clarify the reason behind it.

Organizers
Udo Hertrich-Jeromin (Vienna University of Technology)
Atsufumi Honda (Yokohama National University)
Masatoshi Kokubu (Tokyo Denki University)
Yoshihiro Ohnita (Waseda University and OCAMI, Osaka Metropolitan University)
Gudrun Szewieczek (University of Innsbruck)